ISSN 1066-369X, Russian Mathematics, 2017, Vol. 61, No. 12, pp. 1–12. © Allerton Press, Inc., 2017.
Original Russian Text © A.N. Abyzov, 2017, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2017, No. 12, pp. 3–15.

Rings Over Which Every Module is an I_0^*-Module

A. N. Abyzov[1*]

[1] *Kazan Federal University*
ul. Kremlyovskaya 18, Kazan, 420008 Russia

Received August 23, 2016

Abstract—We obtain a description of semi-artinian rings over which every module is an I_0^*-module. We also describe semi-artinian rings over which every module is a direct sum of a projective module and a V-module.

DOI: 10.3103/S1066369X17120015

Keywords: *semi-artinian ring, V-ring, I_0-module, I_0^*-module, quasiprojective module.*

INTRODUCTION

Throughout the paper we assume that all rings are associative with identity and all modules are unital. For instance, if a ring is both left and right semi-artinian, then we say that the ring is semi-artinian.

Being a continuation of [1], this paper deals with semi-artinian rings over which every right module is an I_0^*-module. A description of these rings is given in the second Section of the paper. The third Section is concerned with studying rings over which every module is a direct sum of a projective module and a V-module. In particular, it follows from the results of the third Section that the following conditions are equivalent for an arbitrary ring R:

1) R is a ring such that every right R-module is a direct sum of a projective module and an SV-module;

2) R is a ring such that every right R-module is a direct sum of an injective module and an SV-module.

It is worth to note that the above statement can be considered as a generalization of the following well-known result.

Theorem ([2], 29.19). *For a ring R the following conditions are equivalent*:

1) *every right R-module is a direct sum of a semisimple module and a projective module*;

2) *every right R-module is a direct sum of a semisimple module and an injective module.*

A generalization of the preceding theorem for an arbitrary Wisbauer category was obtained in [3].

Let M and N be right R-modules. If $N \in \sigma(M)$, then we denote by $E_M(N)$ the injective hull of the module N in the category $\sigma(M)$. We denote by $Z_M(N)$ the greatest M-singular submodule of the module N, i.e., the submodule

$$Z_M(N) = \sum_{f \in \mathrm{Hom}_R(X,N),\ \mathrm{Ker}\, f \trianglelefteq X,\ X \in \sigma(M)} f(X).$$

A module N is said to be *M-singular* (*non-M-singular*) provided that $N = Z_M(N)$ ($Z_M(N) = 0$).

If α is the least ordinal such that $\mathrm{Soc}_\alpha(M) = Soc_{\alpha+1}(M)$, then the submodule $\mathrm{Soc}_\alpha(M)$ is denoted by $L(M)$. Given a ring R the ideal $L(R_R)$ in R is denoted by $L(R)$.

In what follows we make use of the definitions from the book [4] and the standard notions and the notations of ring theory (see, e.g., [5–7]).

[*] E-mail: adel.abyzov@kpfu.ru.